Overview of the Seminar Topic

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Contents

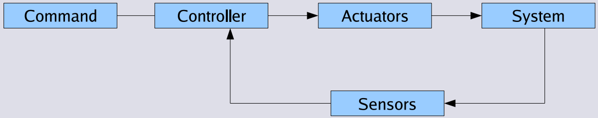
Purpose of Control Theory

Physical System

Control Signals

- Mathematical theory of what kind of control signals we should inject to a physical system in order to reach a goal
- For example, physical system can be a car, which obeys the Newton's laws (or of GR, if desired)
- The control signals are steering of the wheel, brakes and the throttle
- The goal to maintain the car on the road

Building Blocks of Control Systems



- Physical system is the actual system to be controlled, sometimes called the plant
- Controller is the system (e.g., computer program), which computes the required control signal (voltage value, motor speed)
- Actuators are the devices used for realizing the physical control signals (e.g., motors, valves, jet engines)
- Sensors are used for monitoring the state of the physical system (e.g., speedometer, acceleration sensor, odometer)

Example Applications of Control

- Cruise control: Maintain constant speed of a car
- Airplane control: Fly the plane to given altitude
- Spacecraft control: Steer Apollo 11 to the moon with minimum consumed fuel
- Chemical process control: Maintain desired level of a chemical substance
- Nuclear reactor control: Keeping the number of neutrons at suitable level
- Robot control: Drive of a robot from one place to another in minimum time
- Disk drive control: Move reading head over the correct track

Topic of the Seminar

- Topic: Automatic control of uncertain dynamic systems
- Uncertainty in different parts of the system:
  - System dynamics
  - System parameters
  - Sensor measurements
- Stochastic (optimal) control: Uncertainties are modeled as stochastic processes and random variables (Bayesian approach)
- Adaptive control: Adaptation mechanisms are used for making the system behave as it should
- Classical control forms the basis of above methods

History: Prehistory and Primitive Period

- Water clock control systems of ancient Greeks and Arabs
- Windmills controllers, temperature regulators, float regulators
- Watt's flyball governor in 1769 for steam engine control, Polzunov's water level regulator in 1765
- In 1868, Maxwell's mathematical analysis on the governor and Vyshnegradskii's analysis on regulators in 1877.
- In late 1800's, Lyapunov's stability theory (in Russia),
- In late 1800's, Heaviside's operational calculus

History: Classical Period

- Beginning of 1900's: Frequency domain (Fourier transform) analysis of feedback amplifiers by Bode, Nyquist and Black.
- In 1922 Minorsky introduces PID controller
- World War II: Airplane pilots, gun-positioning systems, radar control systems.
- Systematic engineering methods of classical control in 40's.
- In 40's, Wiener's work on stochastic analysis and optimal filtering (and "cybernetics")
- In 50's, s-plane methods (Laplace domain) such as root locus were developed, as well as digital control and the z-transform.

History: Modern Period

- In late 50's, state space models, Bellman's dynamic programming approach to discrete-time optimal control, Pontryagin's calculus of variations approach to optimal control
- In 60's, Kalman's theory of Linear Quadratic Regulators (LQR), Kalman filter and Kalman-Bucy filter. Stability analysis of linear state space models (mostly by Kalman).
- In 60's, non-linear control, stochastic control, adaptive control and numerical methods for computer implementation
- 70's to present: robust control, numerical methods, approximation methods and industrial applications

Classical Control [1/2]

- Systems are modeled with linear time-invariant (LTI) ordinary differential equations (ODE) with single input and single output (SISO), e.g.

$$d^2y(t)/dt^2 + a\,dy(t)/dt + b\,y(t) = u(t).$$

- Based of forming an error signal $e(t) = y(t) - r(t)$ (feedback signal) between reference signal and the process.
- The controller is designed to force the error to zero
- Analysis is typically done using Laplace transforms and s-domain transfer functions of the differential equations

$$H(s) = \frac{1}{s^2 + a\,s + b}$$

Classical Control [2/2]

- The most common control architecture is proportional-integral-derivative (PID) controller

$$C(s) = K_p + K_i/s + K_d\, s$$

- The parameters of controllers are heuristically selected such that the controlled closed loop system is stable
- Performance criteria are, for example, settling time, overshoot, oscillations and robustness properties
- Phase-lead, phase-lag and other compensators can be used for improving robustness

Optimal Control [1/2]

- Systems are modeled generally as non-linear state space models, that is, non-linear vectorial differential or difference equations, e.g.

$$d\mathbf{x}(t)/dt = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$$

- $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))$ is the state vector
- $\mathbf{u}(t) = (u_1(t), \ldots, u_d(t))$ is the control signal
- The controller is selected to optimize a given performance cost functional such as

$$J(\mathbf{u}) = \int_0^T \left[||\mathbf{x}(t) - \mathbf{r}(t)||^2 + ||\mathbf{u}(t)||^2 \right] dt$$

- All classical controllers can be seen to be special cases of optimal controllers

Optimal Control [2/2]

- In continuous-time, the solution is given by the calculus of variations
- In discrete-time case, the problem is a finite-dimensional optimization problem
- The solutions can be expressed in several forms: Euler-Lagrange equations, Pontryagin's maximum principle, Dynamic programming, Hamilton-Jacobi-Bellman equation
- Linear Quadratic Regulator (LQR) is an important closed form solution and can be used for approximating non-linear solutions

Stochastic Control [1/2]

- The system is assumed to contain unknown processes and parameters, which are modeled using stochastic processes and random variables
- Dynamics are modeled using stochastic differential or difference equations

$$d\mathbf{x}(t)/dt = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) + \mathbf{G}(\mathbf{x}(t), t)\,\mathbf{w}(t)$$

- Noise process $\mathbf{w}(t)$ models the uncertainties in dynamics
- System state is not measured directly, but through certain sensors
- Sensor measurements also contain uncertainties or noises that are modeled as stochastic processes and random variables

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), t) + \mathbf{v}(t)$$

Stochastic Control [2/2]

- The performance measure is the minimum expected value of cost function such as

$$J(\mathbf{u}) = E\left\{ \int_0^T \left[||\mathbf{x}(t) - \mathbf{r}(t)||^2 + ||\mathbf{u}(t)||^2 \right] dt \,\middle|\, \mathbf{y}[0,t] \right\}$$

- The solution is a complicated combination of optimal filtering and stochastic generalizations of the optimal control solutions
- The most general solution can be thought as recursive application of Bayesian inference and Bayesian decision theory
- Linear Quadratic Gaussian Regulator is an important closed form solution, which is a combination of Kalman filter and deterministic LQR.

Division to Classical, Optimal and Stochastic Control

- Control approaches can be thought as subsets of each other as follows:

$$\textit{Classical} \subset \textit{Optimal} \subset \textit{Stochastic Control}$$

- This division is similar to the division

$$\textit{Least squares} \subset \textit{Maximum likelihood} \subset \textit{Bayesian inference}$$

- Stochastic control is based on Bayesian decision theory:

$$\textit{Stochastic Control} \subset \textit{Bayesian decision theory}$$

Robust, Adaptive and Intelligent and Other Controls

- Robust control refers methodology for design controllers that are robust to model uncertainty and unknown disturbances
- In adaptive control, model or parameter identification is included in the control system.
- Intelligent control is general name for control methods utilizing methods from machine learning (AI), e.g., neural networks
- Can be thought as applications or approximations of general Bayesian decision theory

Division to Classical and Modern Control

- In 60's, division into classical and modern control
- Classical refers to transfer function based analysis of scalar linear time-invariant systems
- Modern refers to state space model based control, e.g., optimal, robust and stochastic control.
- This division is a bit outdated - if in the 60's control was "modern", what is it today?

Division to Continuous-Time and Discrete-Time

- Physical systems are naturally modeled as continuous-time systems, i.e., with differential equations
- In computer, due to sampling of signal, a reasonable way of modeling is sometimes discrete-time formulation with difference equations
- There are processes that are more naturally modeled with discrete-time than continuous-time models.
- Mathematically every discrete-time process can be represented as sampled version of a continuous-time process.
- Numerically discrete-time processes are easier, because computers work in discrete steps.

Summary

- Control theory studies how a physical system can be steered to a given goal by applying a control force
- Dates back to 1700's and 1800's, but most of the theory was has developed in 1900's
- The main approaches are classical, optimal and stochastic control.
- Robust, adaptive, intelligent and other approaches also exist, but they can be considered as special cases of the main methods.
- The mathematical foundations are in differential equations, Laplace/Fourier transforms, calculus of variations, stochastic analysis and Bayesian decision theory.