Overview of the Seminar Topic

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What is Control Theory?

Purpose of Control Theory

- Mathematical theory of what kind of control signals we should inject to a physical system in order to reach a goal
- For example, physical system can be a car, which obeys the Newton’s laws (or of GR, if desired)
- The control signals are steering of the wheel, brakes and the throttle
- The goal to maintain the car on the road
**Building Blocks of Control Systems**

- **Physical system** is the actual system to be controlled, sometimes called the plant.
- **Controller** is the system (e.g., computer program), which computes the required control signal (voltage value, motor speed).
- **Actuators** are the devices used for realizing the physical control signals (e.g., motors, valves, jet engines).
- **Sensors** are used for monitoring the state of the physical system (e.g., speedometer, acceleration sensor, odometer).
Example Applications of Control

- **Cruise control:** Maintain constant speed of a car
- **Airplane control:** Fly the plane to given altitude
- **Spacecraft control:** Steer Apollo 11 to the moon with minimum consumed fuel
- **Chemical process control:** Maintain desired level of a chemical substance
- **Nuclear reactor control:** Keeping the number of neutrons at suitable level
- **Robot control:** Drive of a robot from one place to another in minimum time
- **Disk drive control:** Move reading head over the correct track
Topic of the Seminar

- **Topic**: Automatic control of uncertain dynamic systems
- **Uncertainty** in different parts of the system:
  - System dynamics
  - System parameters
  - Sensor measurements
- **Stochastic (optimal) control**: Uncertainties are modeled as stochastic processes and random variables (Bayesian approach)
- **Adaptive control**: Adaptation mechanisms are used for making the system behave as it should
- **Classical control** forms the basis of above methods
History: Prehistory and Primitive Period

- **Water clock** control systems of ancient Greeks and Arabs
- Windmills controllers, temperature regulators, float regulators
- Watt’s **flyball governor** in 1769 for steam engine control, Polzunov’s water level regulator in 1765
- In 1868, Maxwell’s **mathematical analysis on the governor** and Vyshnegradskii’s analysis on regulators in 1877.
- In late 1800’s, Lyapunov’s **stability theory** (in Russia),
- In late 1800’s, Heaviside’s **operational calculus**
History: Classical Period

- Beginning of 1900’s: Frequency domain (Fourier transform) analysis of feedback amplifiers by Bode, Nyquist and Black.
- In 1922 Minorsky introduces PID controller
- World War II: Airplane pilots, gun-positioning systems, radar control systems.
- Systematic engineering methods of classical control in 40’s.
- In 40’s, Wiener’s work on stochastic analysis and optimal filtering (and “cybernetics”)
- In 50’s, s-plane methods (Laplace domain) such as root locus were developed, as well as digital control and the z-transform.
History: Modern Period

- In late 50’s, state space models, Bellman’s dynamic programming approach to discrete-time optimal control, Pontryagin’s calculus of variations approach to optimal control
- In 60’s, Kalman’s theory of Linear Quadratic Regulators (LQR), Kalman filter and Kalman-Bucy filter. Stability analysis of linear state space models (mostly by Kalman).
- In 60’s, non-linear control, stochastic control, adaptive control and numerical methods for computer implementation
- 70’s to present: robust control, numerical methods, approximation methods and industrial applications
Systems are modeled with linear time-invariant (LTI) ordinary differential equations (ODE) with single input and single output (SISO), e.g.

\[ \frac{d^2 y(t)}{dt^2} + a \frac{dy(t)}{dt} + b y(t) = u(t). \]

Based on forming an error signal \( e(t) = y(t) - r(t) \) (feedback signal) between reference signal and the process.

The controller is designed to force the error to zero.

Analysis is typically done using Laplace transforms and s-domain transfer functions of the differential equations

\[ H(s) = \frac{1}{s^2 + a s + b} \]
The most common control architecture is proportional-integral-derivative (PID) controller

\[ C(s) = K_p + K_i/s + K_d s \]

The parameters of controllers are heuristically selected such that the controlled closed loop system is stable.

Performance criteria are, for example, settling time, overshoot, oscillations and robustness properties.

Phase-lead, phase-lag and other compensators can be used for improving robustness.
Optimal Control [1/2]

- Systems are modeled generally as non-linear state space models, that is, non-linear vectorial differential or difference equations, e.g.

\[ \frac{dx(t)}{dt} = f(x(t), u(t), t) \]

- \( x(t) = (x_1(t), \ldots, x_n(t)) \) is the state vector
- \( u(t) = (u_1(t), \ldots, u_d(t)) \) is the control signal
- The controller is selected to optimize a given performance cost functional such as

\[ J(u) = \int_0^T \left[ ||x(t) - r(t)||^2 + ||u(t)||^2 \right] dt \]

- All classical controllers can be seen to be special cases of optimal controllers
In continuous-time, the solution is given by the calculus of variations.

In discrete-time case, the problem is a finite-dimensional optimization problem.

The solutions can be expressed in several forms: Euler-Lagrange equations, Pontryagin’s maximum principle, Dynamic programming, Hamilton-Jacobi-Bellman equation.

Linear Quadratic Regulator (LQR) is an important closed form solution and can be used for approximating non-linear solutions.
Stochastic Control [1/2]

- The system is assumed to contain unknown processes and parameters, which are modeled using stochastic processes and random variables.
- Dynamics are modeled using stochastic differential or difference equations:
  \[
  \frac{dx(t)}{dt} = f(x(t), u(t), t) + G(x(t), t) w(t)
  \]
- Noise process \( w(t) \) models the uncertainties in dynamics.
- System state is not measured directly, but through certain sensors.
- Sensor measurements also contain uncertainties or noises that are modeled as stochastic processes and random variables:
  \[
  y(t) = h(x(t), u(t), t) + v(t)
  \]
The performance measure is the minimum expected value of cost function such as

$$J(u) = E \left\{ \int_0^T \left[ ||x(t) - r(t)||^2 + ||u(t)||^2 \right] dt \mid y[0, t] \right\}$$

The solution is a complicated combination of optimal filtering and stochastic generalizations of the optimal control solutions.

The most general solution can be thought as recursive application of Bayesian inference and Bayesian decision theory.

Linear Quadratic Gaussian Regulator is an important closed form solution, which is a combination of Kalman filter and deterministic LQR.
Control approaches can be thought as subsets of each other as follows:

\[ \text{Classical} \subset \text{Optimal} \subset \text{Stochastic Control} \]

This division is similar to the division

\[ \text{Least squares} \subset \text{Maximum likelihood} \subset \text{Bayesian inference} \]

Stochastic control is based on Bayesian decision theory:

\[ \text{Stochastic Control} \subset \text{Bayesian decision theory} \]
Robust, Adaptive and Intelligent and Other Controls

- Robust control refers methodology for design controllers that are robust to model uncertainty and unknown disturbances.
- In adaptive control, model or parameter identification is included in the control system.
- Intelligent control is general name for control methods utilizing methods from machine learning (AI), e.g., neural networks.
- Can be thought as applications or approximations of general Bayesian decision theory.
In 60’s, division into classical and modern control

Classical refers to transfer function based analysis of scalar linear time-invariant systems

Modern refers to state space model based control, e.g., optimal, robust and stochastic control.

This division is a bit outdated - if in the 60’s control was "modern", what is it today?
Physical systems are naturally modeled as continuous-time systems, i.e., with differential equations.

In computer, due to sampling of signal, a reasonable way of modeling is sometimes discrete-time formulation with difference equations.

There are processes that are more naturally modeled with discrete-time than continuous-time models.

Mathematically every discrete-time process can be represented as sampled version of a continuous-time process.

Numerically discrete-time processes are easier, because computers work in discrete steps.
Control theory studies how a physical system can be steered to a given goal by applying a control force.

Dates back to 1700’s and 1800’s, but most of the theory was has developed in 1900’s.

The main approaches are classical, optimal and stochastic control.

Robust, adaptive, intelligent and other approaches also exist, but they can be considered as special cases of the main methods.

The mathematical foundations are in differential equations, Laplace/Fourier transforms, calculus of variations, stochastic analysis and Bayesian decision theory.