Adaptive control of a time-varying system

Version 1.0

1 The system

Consider a system with a changing gain — for example, an truck lift with varying mass and setpoint.

We use force to control so the basic equation is as follows.

$$\ddot{x} = K_r u + R(0, 1, 1)$$

where $R(\mu, \sigma, \lambda)$ is the Ornstein-Uhlenbeck process (mean-reverting process) with the given mean, volatility and reversion rate.

The process gain K_r is a random process defined as follows

$$K_r = D_{P(t,0.05)}$$

where $P(t, \lambda)$ is a Poisson process with rate λ and D_i for $i \geq 0$ are i.i.d. random variables whose logarithm is uniformly distributed: $\log D_i = U(\log 0.25, \log 32.0)$ between the logarithms of 0.25 and 32.0. This results in a distribution where the density is inversely proportional to the value. This is very important for the results below, because different distributions would emphasize either small or large values of K_r too much and allow compromises to be made easier in a controller (another alternative would be to penalize large deviations from the setpoint by a more severe factor than square).

In a simulation, this means that a new value is selected from the uniform distribution at each timestep with the probability $0.05\Delta t$, and the exponential of this value is used as the new process gain.

Similarly, the setpoint x_r is defined by

$$x_r = E_{P(t,0.2)}$$

where E_i for $i \geq 0$ are i.i.d. random variables with the uniform distribution $U(0, 5)$.

We shall clamp the system so that the velocity $\dot{x}$ and location x cannot go outside the interval $[-100, 100]$. To make things simple we shall do this after integration, so that the velocity used and measured will not be affected by the clamping, i.e., to obtain velocity, integrate acceleration in time step by step, and at each step, if the integrated value is outside the interval, return it to be within the interval.

Simulate the system using $dt = 1/1024$, with an additional 1/8 time unit delay in the control signal (delay 0 would imply that the control could use the results of the previous cycle so in a way the delay is $1024/8 + 1 = 129$ cycles).

The observations are x and $\dot{x}$, but to each a separate Ornstein-Uhlenbeck process $R(0, 0.5, 1)$ is added when the observation is given to the controller.

The value function of the system is

$$\int (x - x_r)^2 dt$$

so using the control signal is not penalized. Note that it is not optimal to use very large controls because the observations are noisy.

Fig. 1 shows an example of the system running on a fixed PID controller.

2 The assignments

1. Simulate the system while fixing K_r to 1. and find a reasonable PD controller for the system. Since there are only two parameters, an exhaustive search (using a contour graph) is possible. Calculate the norm for a reasonable length of time, say 256 or 512 seconds. If everything goes all right, you should be able to get the norm per time to be slightly under 0.5.

2. Show that when K_r is not fixed, this controller does very badly, and in fact, that there is no good PD controller for this system, as the gains either make the system fluctuate when there K_r is high or make it to sluggish when K_r is small.

3. Assume that we are able to observe an additional output from the process giving the process gain plus some Ornstein-Uhlenbeck noise: $K_r + R(0, 0.1, 1)$. Build a controller using gain scheduling and evaluate its performance relative to the controllers created in tasks 1) and 3).

4. Develop an adaptive controller in the situation where there is no extra observation. For example, adding a sine wave into the control signal and observing that frequency in the output signal can tell you about the gain. Evaluate the performance. Describe your controller in terms of the ones you have learned about in the course.

5. For 4) you most likely chose a controller that is simple to implement. If you had more time and the optimizing the value function would be important, which control methods would you consider applying to this problem? Justify your choices.

For assignments 3) and 4) above, the results from different people attending the course will be tabulated and compared so do your best.

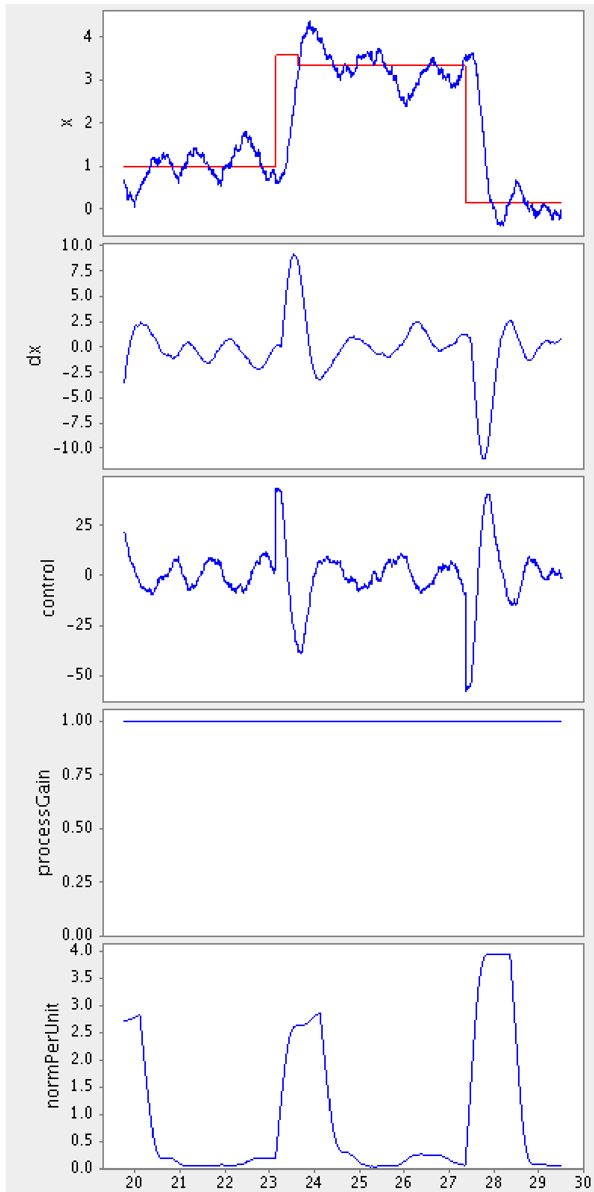

Figure 1: The system with the process gain fixed to unity being run with a PID controller with $K_P = 15$ and $K_D = 5$ (which is not the best possible one). From top to bottom the graphs show the current location (including the setpoint with red), the velocity, the control signal, the process gain, and finally the integral of the norm in the previous one time unit (which is useful for debugging but not for evaluating the systems - for that, the integral must be taken over all time).