S-114.4150 Complex Networks, autumn 2006

Problem set #2 (due Thu. 16.11.)

1. Show that the “global” clustering coefficient $C_g$ of a random graph defined by

$$C_g = \frac{3E(\# \text{ of triangles})}{E(\# \text{ of 2-stars})}$$

(1)

can be “localized” with the following definition:

$$C_{loc} = E(C_i), \quad C_i = \frac{E(\# \text{ of triangles containing } i)}{E(\# \text{ of 2-stars centered on } i)}.$$  

(2)

Calculate $C_g$ and $C_{loc}$ for the Erdős-Rényi network.

2. Derive the clustering coefficient of a random graph with specified degree distribution (i.e. the expected number of links between two neighbours of a randomly chosen vertex) as a function of $N$ and $\langle k \rangle$.

3. Derive the clustering coefficient of a network as a function of its adjacency matrix.

4. Consider a small-world network that consists of a regular 1D lattice with periodic boundary conditions and shortcuts. Let $r$ be the maximum range of edges in the underlying lattice and $p$ the probability per edge on the underlying lattice of adding a shortcut (i.e. the expected number of shortcuts is $Nr p$). Derive the clustering coefficient of this network using the definition

$$C := \frac{3E(\# \text{ of triangles})}{E(\# \text{ of 2-stars})}.$$  

(3)

You may assume that $N$ is large and $r << N$. 