

---

**S-114.100 Computational Science / Laskennallinen tiede. Fall 2003.**

Assignment 9. Monte Carlo -simple sampling.

Lecture notes 9.

Due Thu 24.11.2004 (5 problems, total of 5 points)

Web page: [www.lce.hut.fi/teaching/S-114.100/](http://www.lce.hut.fi/teaching/S-114.100/)

RNGs: [www.lce.hut.fi/teaching/S-114.100/codes/](http://www.lce.hut.fi/teaching/S-114.100/codes/)

*computer* = programming task (C / C++ / Fortran / Java)

*pencil and paper* = solve on paper

---

**Problem 1.** (*pencil and paper / computer*) (2 points)

(a) Suppose that you use the Monte Carlo method to estimate the value of a given quantity  $A$ . One simulation run gives you a single numerical value denoted by  $A_i$ . How can you obtain a reliable estimate of the true value of  $A$  and how can you calculate an estimate of the error?

(b) Write a program that computes an estimate of  $\pi$  using the "hit-or-miss" Monte Carlo method. The program should also calculate an error estimate  $\sigma$  (as you described in part (a)) and average deviation from the correct value ( $\pi = 3.141592654$ ).

Do this by generating  $N$  uniformly distributed random points inside the square defined by  $-R \leq x \leq R$  and  $-R \leq y \leq R$ . Calculate the number of points which are inside the circle  $x^2 + y^2 = R^2$ . For each value of  $N$ , perform  $n = 1000$  independent measurements. The resulting average value

$$\pi_{est} = \langle \pi \rangle = \frac{1}{n} \sum_{i=0}^n \pi_i$$

is your MC estimate. The average deviation from the correct value is given by

$$\frac{1}{n} \sum_{i=0}^n |\pi_i - \pi|$$

Plot your estimate of  $\pi$ , the error estimate  $\sigma$  and the average absolute error as a function of  $N$  for  $N = 1000 - 30000$  (e.g. do a series of runs increasing  $N$  by 1000 at each round).

What relation does the error estimate follow? What about the absolute error?

**Problem 2.** (*computer*)

Write a program which uses the "sample-mean" method to compute the integral

$$I = \int_0^2 \left\{ \int_3^6 \left[ \int_{-1}^1 (yx^2 + z \ln y + e^x) dx \right] dy \right\} dz$$

Include the calculation of an error estimate in your program. Show the results as a function of  $N$  (number of random points). The correct answer is  $I \approx 49.9213$ .

**Problem 3** (*computer*)

(a) Write a program which uses importance sampling to compute the integral

$$I = \int_1^2 (2x^8 - 1) dx$$

Plot the result as a function of  $N$ .

(b) Compute the same integral using the sample mean method. Plot the absolute error  $|I_{est} - I|$  as a function of  $N$  for the results obtained using the sample mean method and for the results obtained in (a) using importance sampling. Which method is better?

**Problem 4.** (*computer*)

Write a program which simulates the process of radioactive decay. You are given a sample of  $N = 20000$  radioactive nuclei each of which decays at a rate  $p$  per second. What is the half-life of the sample if  $p = 0.4$ ?

Calculate the estimate of the half-life  $\langle t_{1/2} \rangle$  from  $m$  independent measurements and include an estimation of the error in your program. Increase  $m$  until you reach an accuracy of at least 0.005.