
S-114.100 Computational Science / Laskennallinen tiede. Fall 2004.

Assignment 6. Splines.

Chapter 6 in Lecture notes.

Due Thu 27.10.2003 (4 problems, total of 5 points)

Web page: www.lce.hut.fi/teaching/S-114.100/

computer = programming task (C / C++ / Fortran / Java)

pencil and paper = solve on paper

Problem 1. (*pencil and paper*)

Determine the natural cubic spline that interpolates the function $f(x) = x^6$ over the interval $[0, 2]$ using knots 0, 1 and 2.

Problem 2. (*computer*)

Write a program which determines a *natural cubic spline* $S(x)$ based on a table of values and evaluates the spline function at a given value of x . Test the code by determining the natural cubic spline interpolant for $f(x) = \sin(x) + 0.04x^2$ at the following 15 *unevenly* spaced knots in the interval $[-9.7, 12.3]$:

$x_0 \cdots x_4$	-9.7000	-7.3000	-5.4000	-5.0000	-3.0100
$x_5 \cdots x_9$	-2.1300	-1.2000	-0.5600	0.0000	1.2000
$x_{10} \cdots x_{14}$	4.5000	6.7000	9.9000	10.0000	12.3000

Evaluate the spline at 100 equally spaced points in the interval $[-9.7, 12.3]$. Print out the value of $S(x)$ and also the absolute error $|S(x) - f(x)|$. Present your results graphically ($S(x)$ and $f(x)$ vs. x and $|S(x) - f(x)|$ vs. x).

Problem 3. (*computer*)

Write a program to estimate $f'(x)$ for any x in $[a, b]$ assuming that we know only the values of $f(x)$ at knots $a = t_0 < t_1 < \dots < t_n = b$. Test your program using the set of data points $\{x, f(x)\}$ given in Problem 2 (use the given x values and the corresponding values of $f(x) = \sin(x) + 0.04x^2$).

Evaluate the derivative of the spline function $S'(x)$ at 100 equally spaced points in the interval $[-9.7, 12.3]$. Present your results graphically in a plot showing $S'(x)$ and $f'(x) = \cos(x) + 0.08x$ as a function of x .

Problem 4. (*computer* / MATLAB) (2 points)

(a) Use your programs developed in Problems 2 and 3 to determine the natural cubic interpolant $S(x)$ and the derivative $S'(x)$ for a set of experimental data that can be downloaded from the course website:

<http://www.lce.hut.fi/teaching/S-114.100/titanium.dat>

Present your results graphically (plot of $S(x)$ vs. x and $S'(x)$ vs. x).

(b) Examine what functions can be found in the Matlab Spline toolbox. Repeat part (a) using Matlab functions. Compare the results. Are there differences? What is causing them?