S-114.100 Computational Science / Laskennallinen tiede. Fall 2004.

Assignment 5. Systems of linear equations.
Chapter 5 in Lecture notes.
Due Thu 27.10.2003 (3 problems, total of 5 points)

Web page: `www.lce.hut.fi/teaching/S-114.100/`

computer = programming task (C / C++ / Fortran / Java)
pencil and paper = solve on paper

Problem 1. (*pencil and paper*)
Solve (on paper)

$$\begin{cases} \varepsilon x_1 & + & x_2 & + & x_3 & = & 5 \\ x_1 & + & x_2 & & & = & 3 \\ x_1 & & & + & x_3 & = & 4 \end{cases}$$

at the limit $\varepsilon \to 0$.

(a) Use basic Gaussian elimination.

(b) Use Gaussian elimination with partial pivoting.

Problem 2. (*computer*) (2 points)
Write two programs that solve a set of linear equations, one using basic Gaussian elimination and the other using Gaussian elimination with partial pivoting. The task is to measure the CPU time as a function of the number of variables N. For this purpose, you can obtain the system of N linear equations by using random numbers as the elements of the arrays A and b.

Present your results graphically and determine how the methods scale.

Hints:

How to generate random numbers? In C/C++ you can use `double drand48(void)` defined in `stdlib.h`, in Fortran the intrinsic function `random_number(harvest)`, and in Java the library method `static double Math.random()`. All functions return uniformly distributed random numbers between 0 and 1.

How to measure CPU time usage? The easiest way is to put command `time` before the actual command (e.g. `time a.out`). See `man time` for details.

Problem 3. (*computer / pencil and paper*) (2 points)
The Hilbert matrix of order n is defined by

$$a_{ij} = (i + j - 1)^{-1} \qquad \text{for } 1 \leq i, j \leq n$$

Define

$$b_i = \sum_{j=1}^{n} a_{ij}$$

The correct solution of the system of equations $\sum_{j=1}^{n} a_{ij} x_j = b_i$ for $1 \leq i, j \leq n$ is given by $\mathbf{x} = [1, 1, \ldots, 1]^T$.

(a) Use your program for Gaussian elimination with partial pivoting (from the previous problem) on this system for a range of n values: $2 \leq n \leq 15$. Print out the results.

(b) Solve the case $n = 3$ by hand using only 3 significant digits for the calculations. Use the results to explain what causes the difficulties observed in the numerical results of part (a).