

---

**S-114.100 Computational Science / Laskennallinen tiede. Fall 2004.**

Assignment 3. Polynomial interpolation.

Chapter 3 in Lecture notes.

Due Thu 13.10.2003 (4 problems, total of 5 points)

Web page: [www.lce.hut.fi/teaching/S-114.100/](http://www.lce.hut.fi/teaching/S-114.100/)

*computer* = programming task (C / C++ / Fortran / Java)

*pencil and paper* = solve on paper

---

**Problem 1.** (*pencil and paper*)

(a) Use the *Lagrange* interpolation process to obtain a polynomial of least degree that interpolates the following table of values:

$x$	0	2	3	4
$y$	8	14	47	116

(b) Find the *Newton* form of the interpolating polynomial for the same table of values. Check that the two polynomials, the Lagrange form and the Newton form, are identical.

**Problem 2.** (*computer*)

Write a program that interpolates the function  $f(x) = e^x$  on the interval  $[0, 2]$  by a polynomial of degree 10. You should first select eleven nodes on the interval  $[0, 2]$  and use the procedure `Coefficients` (see Lecture notes) to construct the polynomial. Then the procedure `Evaluate` should be used to evaluate the polynomial at 101 points and the result should be compared to the actual value of the function  $f$ . Present the results in graphical form: plot  $p(x)$  and  $f(x)$  vs.  $x$ , and in another plot,  $|p(x) - f(x)|$  vs.  $x$ .

**Problem 3.** (*pencil and paper*)

Given the data

$x$	0	2	4	7	10
$f(x)$	1.000	2.718	7.389	33.12	148.4

do the following: (a) Construct the divided-difference table. (b) Write the Newton's interpolation polynomial in the nested form, and use it to find an approximation to  $f(4.2)$ .

(c) Use the computer program of Problem 2 to check how the divided-differences table is implemented in the procedure `Coefficients` and how the procedure `Evaluate` evaluates the polynomial (e.g., print out some intermediate values).

**Problem 4.** (*computer*) (2 points)

Consider the following *serpentine curve*:

$$f(x) = \frac{x}{1/4 + x^2}$$

(a) Modify the program of Problem 2. (Newton form) to find a polynomial that interpolates the serpentine curve on the interval  $[-2.02857, 2.02857]$ . Use 13 equidistant nodes. Evaluate the polynomial and the value of  $f(x)$  at 101 points on the given interval.

(b) Instead of having a set of equidistant nodes, we can use so-called *Chebyshev nodes* which for the interval  $[-1, 1]$  are given by:

$$x_i = \cos \left[ \left( \frac{i}{n} \right) \pi \right] \quad (0 \leq i \leq n)$$

The corresponding set of nodes on an arbitrary interval  $[a, b]$  is given by

$$x_i = \frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \left[ \left( \frac{i}{n} \right) \pi \right] \quad (0 \leq i \leq n)$$

Modify your program to find a polynomial that interpolates the serpentine curve using 13 Chebyshev nodes on the interval  $[-2.02857, 2.02857]$ . Evaluate the polynomial at 101 equidistant points. Compare to results obtained in part (a). Are the results satisfactory?