S-114.100 Computational Science / Laskennallinen tiede. Fall 2003.

Assignment 2. Locating roots of equations.
Chapter 2 in Lecture notes.
Due Wed 6.10.2003 (3 problems, total of 5 points)

Web page: `www.lce.hut.fi/teaching/S-114.100/`

computer = programming task (C / C++ / Fortran / Java)
pencil and paper = solve on paper

Problem 1. (*computer*)
Write a program which uses the *bisection method* for locating roots of equations. Use the program to find the root of the following equation

$$9x^4 + 18x^3 + 38x^2 - 57x + 14 = 0$$

in the interval $[0, 1]$.

Problem 2. (*computer*) (2 points)
(a) Write a program which uses the *Newton's method* for locating roots of equations. Use the program to find the root of the following equation

$$x^3 - x - 5 = 0$$

Use the initial point $x_0 = 0.57735$. Limit your iterations to 50. Print out the results and explain them.

(b) Construct a hybrid method which utilizes a combination of the bisection and Newton's algorithms to ensure global convergence. This algorithm takes a bisection step whenever Newton's algorithm would take the solution out of bounds. As input, this method needs two numbers a_1 and a_2 which bracket the root and the starting point for the Newton's method x_0 (this can, of course, be computed from the brackets, but for testing purposes use the value $x_0 = 0.57735$).

Hint: Modify your code from part (a) by inserting a condition to use bisection if Newton is out of range.

Problem 3. (*computer / Matlab*) (**2 points**)
Basin of attraction. Consider the complex polynomial $z^3 - 1$. Its roots are the three cube roots of unity. Generate a picture showing the three basins of attraction in the square region defined by $-1 \leq \text{Real}(z) \leq 1$ and $-1 \leq \text{Imag}(z) \leq 1$.

To do this, use a mesh of 1000×1000 pixels inside the square. The center point of each pixel is used to start the iteration of Newton's method. Assign a particular basin color to each pixel if convergence to a root is obtained with a maximum of 100 iterations.

In order to limit the large number of iterations use a criterion for stopping the iteration when it gets within a certain neighborhood of the root. The criterion for convergence is to check both $|z_{n+1} - z_n| < \varepsilon$ and $|z_{n+1}^3 - 1| < \varepsilon$ with a small value such as $\varepsilon = 10^{-12}$ as well as using a maximum number of steps.

Hint: It is best to test your program and to get a crude picture with only a small number of pixels such as 10×10.

Note: For this problem, it is permitted to use only Matlab (C/Java solutions are of course also accepted).