Improved Rotational Invariance for Statistical Inverse in Electrical Impedance Tomography

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Abstract

In this paper we show that rotational invariance can be improved in a neural network based EIT reconstruction approach by a suitably chosen permutation of the input data. The input space is partitioned to non-overlapping sectors, and the input signal is permuted so that it lies in one sector independent of the original rotation angle. We demonstrate the advantages of the method with computer simulations. The proposed approach yields better results in the inverse problem, and allows use of smaller networks with fewer training samples.

1 Introduction

Electrical Impedance Tomography (EIT) is a typical tomographic imaging method, where internal structure of an object, in this case impedance distribution (reciprocal of conductivity), is reconstructed from measurements from the surface. In EIT small alternating current is injected through the object and the resulting potentials are measured with electrodes attached to the surface of the object. The forward problem, or computing the potential field given the conductivity distribution in the object, is highly non-linear when the conductivity level has large variations. Consequently, the inverse problem is severely ill-posed, and requires efficient regularization. See [1] for review and some results.

Recently a novel statistical inverse approach for the EIT reconstruction problem was proposed by one of the authors [2], which is based on transforming the problem to a lower dimensional eigenspace and approximating the inverse mapping with a neural network. We have used Bayesian methods and early stopping committees for estimating the conditional expectation of the conductivity distribution given the measured potential signals. The results so far have suggested that the approach can provide similar or better performance than state-of-the-art inverse methods with several orders of magnitude faster reconstruction.

A central issue in the proposed approach is that the system learns to solve a strongly constrained part of the generic EIT solution domain [3], somewhat analogously to regularization methods where the solution is constrained by smoothing priors.

A specific problem in the approach is rotational invariance. For rotationally symmetric geometry standard inverse methods produce the same result for a given conductivity image independent of its rotation. In the proposed approach in [2] the system learns the inverse mapping for all rotation angles separately. In theory this does not make the problem more difficult to learn, as the different rotation angles occupy different regions in the input and output signal space. However, the statistical estimation task becomes harder, as the redundant degrees of freedom in the problem require respectively more free parameters in the model. The required additional training samples can be generated by rotations, but the actual learning task becomes computationally much more expensive.

Fig. 1 shows an example, where the task is to construct an image of the bubble distribution within a liquid medium inside an industrial pipe. The bubbles have different conductivity levels than the main substance in the pipe. There are 16 electrodes equally spaced on the outer surface. From these each is used for injecting the current, the adjacent electrode is grounded, and the potentials of others are measured resulting in 256 measurements. Note that part of these measurements are still redundant, and are not measured in the actual EIT systems.

As demonstrated in Fig. 1 the rotation of the distribution around the center of the pipe corresponds a permutation of the measurement indices. In the left image in Fig. 1 there is a bubble configuration where the electrode 14 is used

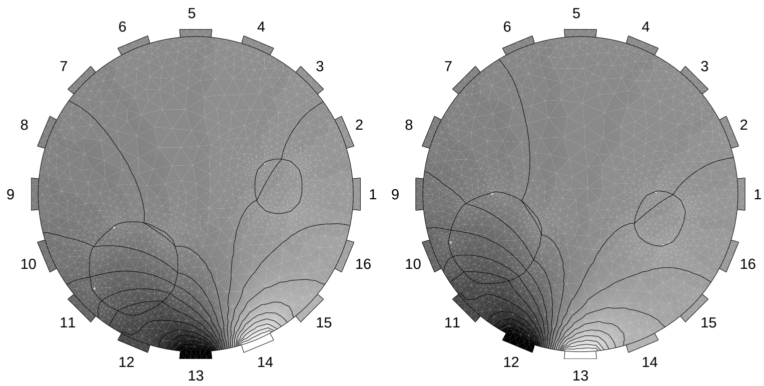

Figure 1: Rotational invariance in EIT: injection from electrode 14 (left image) is equal to injection from electrode 13 of the rotated bubble (right image). The potential field is indicated by the gray level in the images.

for injection. Identical but cyclically shifted electrode potentials are obtained for the rotated configuration on the right with injection from electrode 13.

In this paper we propose a method for improving the rotational invariance by partitioning the signal space to non-overlapping partitions. The data is permuted to always lie in one sector, and the inverse mapping is learnt only there.

2 Rotation Invariant Functions

Definition 1 *A function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is rotationally invariant with a permutation operator σ, with cycle length n, iff for all x there exists a dual operator $\tau : \mathbb{R}^m \longrightarrow \mathbb{R}^m$ such that*

$$f(\sigma x) = \tau f(x). \tag{1}$$

In EIT inverse problem the stacked potential signal is denoted by x (e.g. 256×1 vector formed by stacking the 16 measurement vectors with different injections) and the reconstructed stacked image by $f(x)$. Here τ is the rotation matrix for angle $2\pi/16$ and σ is the corresponding permutation operator of the potential signals x. The permutation is composed of cyclic rotations of each of the measurement vector blocks of size 16, thus σ is formed as

$$\sigma = \sigma_1 \ldots \sigma_{16}, \tag{2}$$

where each σ_i is a cyclic permutation of one block of x. The σ_i and therefore σ has period 16, which is less than the dimension of the input but this is not an obstacle.

2.1 Partition of $\mathbb{R}^n$ for Rotational Invariance

We construct a partition of $\mathbb{R}^n$ to disjoint convex sets $\bigcup_{i=0}^{n-1} \mathcal{P}_i = \mathbb{R}^n$ and $\forall i, j, i \neq j : \mathcal{P}_i \cap \mathcal{P}_j = \emptyset$, such that

$$x \in \mathcal{P}_k \Longleftrightarrow \sigma^{-k} x \in \mathcal{P}_0. \tag{3}$$

Now the inversion mapping need to be modelled only in the sector $\mathcal{P}_0$, and the actual image $f(x)$ is recovered as $\tau^k f(\sigma^{-k} x)$. The data occupies now a more compact space and the inverse system is guaranteed to satisfy definition Def. 1.

2.2 Determination of the Partition

The partition can be determined by some vector $\xi \in \mathbb{R}^n$ as follows:

$$x \in \mathcal{P}_k \Longleftrightarrow \forall l \neq k : \ \xi^T \sigma^k x \geq \xi^T \sigma^l x. \tag{4}$$

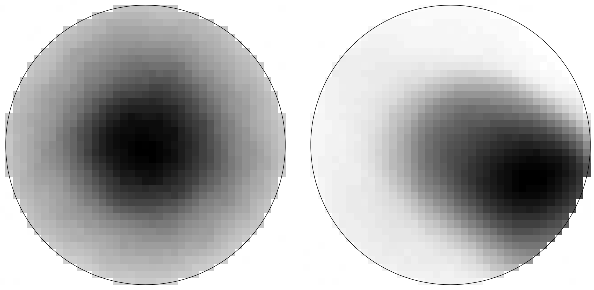

Figure 2: Average over the conductivity distribution of the unrotated (left) and rotated (right) data.

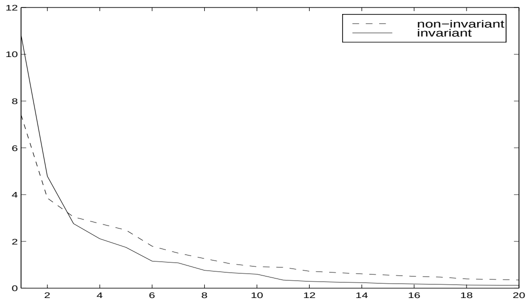

Figure 3: 20 first principal eigenvalues of the potential signals.

Method	MPE %	PCA %
Non-invariant network	10	73
Invariant network	9.1	89

Table 1: Performance of the proposed method in detection of bubbles, averaged over test set of 2800 samples. Mean Pixel Error (MPE%) measures the percentage of image pixels erroneously classified as bubble or background. The percentual principal eigenvalue efficiencies (PCA %) measure the variance in potential signals captured by the first 10 eigenvectors.

Sets $\mathcal{P}_k$ in Eq. 4 are convex. For the sets to be disjoint (except on the borders, which have no significance here), the permutation cycle of ξ must not be smaller than n, or $\sigma^k \xi = \xi$ must not hold for any $k < n$. With, e.g., $\xi = (1, 0, 1, 0, \ldots)^T$ the partition contains only two half spaces, with all the sets $\mathcal{P}_k$ overlaid on them alternately.

3 Simulation Results

The data used in the experiments was generated by drawing a random dataset of size 2800 of overlaid circular bubbles, with varying impedance levels, and computing the corresponding potential measurements with finite element method (FEM). The signals were projected to the first 10 principal components as in [2]. An early-stopped committee of MLP networks was then trained for this data.

3.1 Optimizing the Partition

The vector ξ defining the partition was numerically optimized by using the following target function J

$$J = \frac{\sum_{i=1}^{10} e_i}{\sum_j e_j}, \tag{5}$$

where e_i is the i^{th} eigenvalue of the correlation matrix of the permuted potential signals. The target function measures the ratio of variation in x that can be explained by the first 10 principal components, leading to as compact representation for the input as possible.

3.2 Results

To demonstrate the increased compactness of the problem space, the mean over the bubble conductivity distributions for the original and rotated data are shown in Fig. 2. Clearly the method moves the bubbles systematically to a certain region. Fig. 3 shows the efficiency of the PCA projection for the original potential signal data and the permuted data, indicating that fewer eigenvectors are sufficient for the same representation accuracy.

The inverse problem in EIT is known to be ill-posed and thus noise in the input can have a large effect on the reconstruction result. The robustness of the proposed method against noise was tested by adding random noise to the input and recording the resulting rotations k in Eq.3. The confusion matrix in Fig. 4 shows that the noise does not

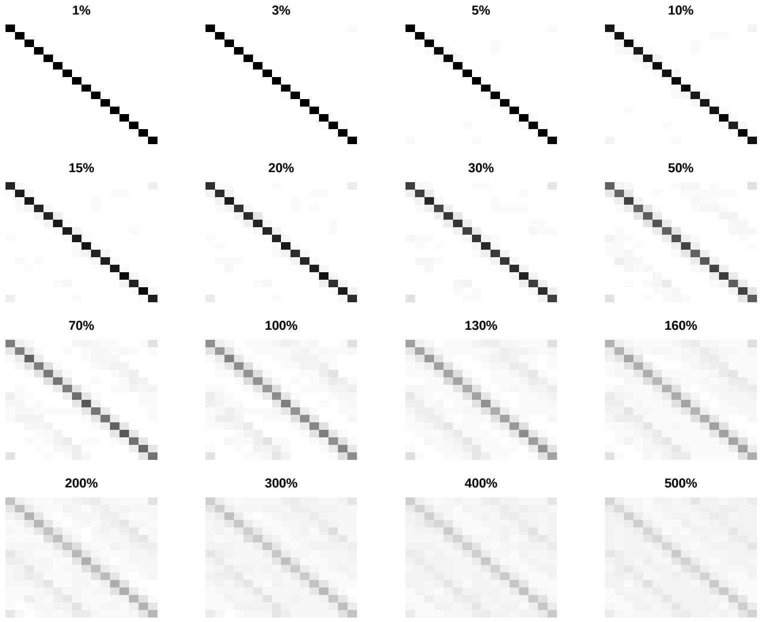

Figure 4: The confusion matrices of partition. The columns correspond to the angle k for noiseless data and rows correspond to the angles due to noisy signals. The noise levels are shown on top of each image.

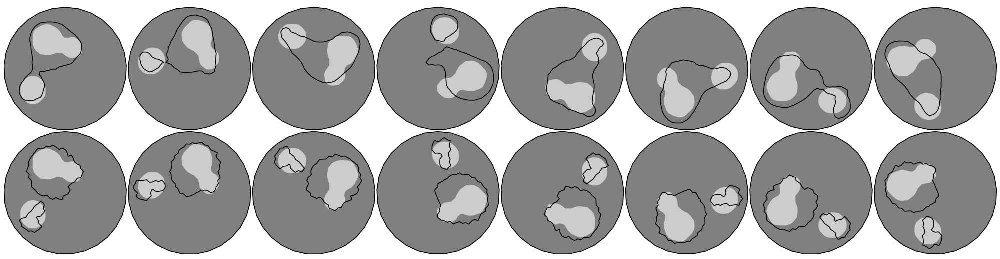

Figure 5: Example of detection of a rotating bubble with the non-invariant (top row), and invariant networks (bottom row), with conductivity level 0.05. Note that in the upper row the result has large variations as the model uses different statistical approximation for each rotation.

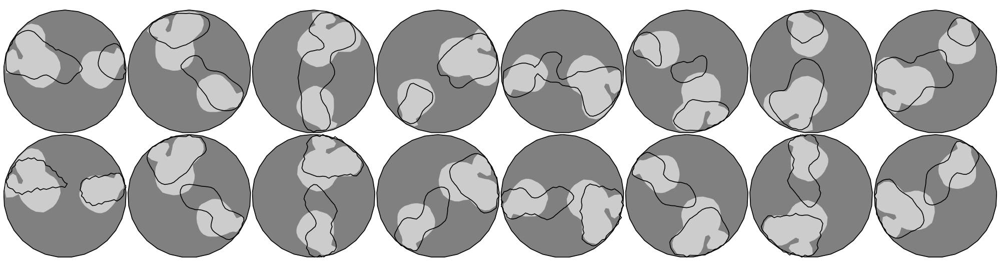

Figure 6: Example of detection of a rotating bubble with conductivity level 0.1. See Fig.5 for explanation.

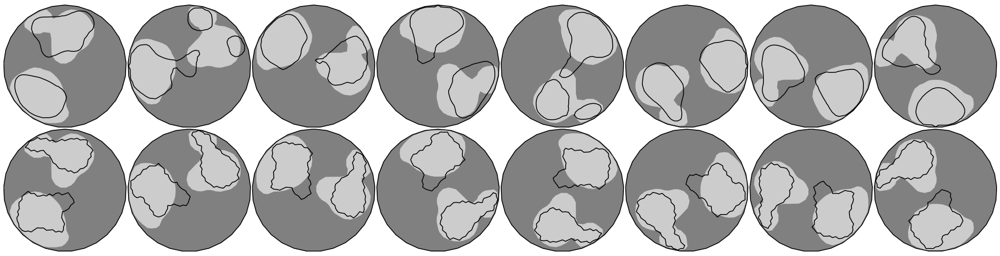

Figure 7: Example of detection of a rotating bubble with conductivity level 0.3. See Fig.5 for explanation.

very significantly affect the rotation angle. With high noise levels the off-diagonal lines indicate symmetric bubble structures for which angles 0 and 8 are about equal.

From Figs. 5, 6, 7 it can be seen that the reconstruction of the non-invariant network is dependent on the angle, whereas the invariant net does not exhibit such behaviour. The fluctuation of the noninvariant system is noticable but not statistically significant, which can be seen from the percentual error which is only 1% -unit greater for the non-invariant network. The ripple on the edges of the invariant network is here due to rotation of the pixels which can be fixed.

Table 1 shows the mean square error and the relative sum of the 10 first principal eigenvalues for the non-invariant and invariant networks. The classification error of the rotation invariant system was observed to be about 1% -unit less than for the uniform system. The invariance also significantly helps the principal value analysis.

4 Conclusions

We have demonstrated that rotational invariance can be improved in neural network EIT reconstruction approach by permuting the input data so that the permutation result, fed into the network, is independent of the original rotation angle of the signals. The performance of the method is tested with simulated data and is shown to be better than that of the non-invariant network. The approach allows use of smaller networks and training data samples, facilitating real-time monitoring systems with moderate computational power in industrial applications.

References

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